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The first integrals and closed-form solutions of optimal control problems

The Pontrygin's maximum principle (Pontryagin, 1987) provides the necessary conditions for the optimum in the optimal control problems in terms of variables time t, state variables q^i, costate variables p_i and control variables u_i. One can eliminate the control variables in terms of state and co-state variables which reduces the conditions of Pontrygin's maximum principle to following non-standard Hamiltonian system:

$$\dot{q}^i = \frac{\partial H}{\partial p_i}, \dot{p}^i = -\frac{\partial H}{\partial q_i} + \Omega^i(t, q^i, p_i.)$$

This type of non-standard Hamiltonian system arises widely in optimal control problems in different fields of the applied mathematics. A mechanical system with non-holonomic nonlinear constraints and non-potential generalized forces results in a non-standard Hamiltonian system. In optimal control problems of economic growth theory involving a non-zero discount factor these type of system arise and are known as a current value Hamiltonian systems. It is proposed how to modify the partial Hamiltonian approach proposed earlier for the current value Hamiltonian systems arising in economic growth theory Naz et al 2014 in order to apply it to the epidemics, mechanics and other areas as well. To show the effective of the approach developed here, it is utilized to construct the first integrals and closed form solutions of some models from real world. Moreover, the essential aspects of infectious diseases spread are uncovered and polices are provided to public health decision makers to compare and implement different control programs. For the Economic growth model some policies are provided to the government in order to have a sustainable growth.