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Shedding vertices and well-covered graphs

A set S of vertices in a graph G is *independent* if no two vertices from S are adjacent. If all maximal independent sets are of the same cardinality, then G is *well-covered* (or *unmixed*) (Plummer, 1970). G belongs to class $\mathbf{W}_2$ if every 2 disjoint independent sets are included in 2 disjoint maximum independent sets (Staples, 1975). There are deep interactions between shellability, vertex decomposability and well-coveredness (Castrillón, Cruz, Reyes, 2016).

Let $v \in V(G)$ and $N(v)$ be its neighborhood. If for every independent set S of $G - (N(v) \cup \{v\})$, there is some $u \in N(v)$ such that $S \cup \{u\}$ is independent, then v is a *shedding vertex* of G (Woodroofe, 2009). Let $Shed(G)$ denote the set of all shedding vertices. Clearly, no isolated vertex is shedding, and no graph in $\mathbf{W}_2$ has isolated vertices.

In this talk, we show that deleting a shedding vertex does not change the maximum size of a maximal independent set including a given independent set. Specifically, for well-covered graphs, it means that a non-isolated vertex $v \in Shed(G)$ if and only if $G - v$ is well-covered. Thus G belongs to class $\mathbf{W_2}$ if and only if $Shed(G) = V(G)$.

There exist well-covered graphs without shedding vertices; e.g., C_7. On the other hand, there are non-well-covered graphs with $Shed(G) = V(G)$.

Problem 1. Find all well-covered graphs having no shedding vertices.

Problem 2. Find all graphs having $Shed(G) = V(G)$.