S. Kobayashi coined the term hyperbolic for a compact complex manifold M without nontrivial holomorphic images of C and conjectured the positivity of the canonical bundle of M. In particular M would be projective if true. But the conjecture is still wide open for projective manifolds beyond dimension two.

A spectacular advance in this direction is the resolution in the projective case by D. Wu-S.T. Yau (Invent. 2016) of the differential geometric analog of the conjecture, due to S.T. Yau. The analog pertains to compact Kahler manifolds with negative holomorphic curvature and the said advance resolves in particular the abundance conjecture, a key conjecture for the classification of algebraic varieties, for such a manifold.

In this talk, I will mainly focus on a recent joint paper with G. Heier, B. Wong and F.Y. Zheng that offers a structure theorem for projective Kahler manifolds with negative holomorphic curvature, assuming the abundance conjecture. The analysis involves a careful study of the rank of the said curvature, and offers relationships to the global abundance problem.