We address the problem of optimal experimental design (OED) for large-scale Bayesian linear inverse problems governed by PDEs. Specifically, the goal is to find optimal placement of sensors where observational data are collected. We focus on sensor placements that maximize the expected information gain. That is, we rely on a Bayesian D-optimal criterion, given by the expected Kullback-Leibler divergence from prior to posterior. In the infinite-dimensional Hilbert space setting, assuming a Gaussian prior and an additive Gaussian noise model, the analytic expression for the OED objective function is given by the log-determinant of a perturbation of the identity by a prior-preconditioned data misfit Hessian operator. We introduce efficient methods to make the computation of OED objective function and its gradient computationally tractable. Numerical results illustrating our framework will be provided in the context of D-optimal sensor placement for optimal recovery of initial state in a time-dependent advection-diffusion problem.