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The combinatory of the arithmetical structures of the path

Given a graph $G = (V, E)$, an arithmetical structure (AS) on G is a pair $(\mathbf{d}, \mathbf{r}) \in \mathbb{N}_+^V \times \mathbb{N}_+^V$ such that $\mathbf{r}$ is primitive and

$$(\mathrm{diag}(\mathbf{d}) - A(G))\mathbf{r}^t = \mathbf{0}^t,$$

where $A(G)$ is the adjacency matrix of G. The concept of AS was introduced by Lorenzini as some intersection matrices that arise in the study of degenerating curves in algebraic geometry. To each AS can be associated a binomial ideal, which recently have been great interest.

In this talk we will give an explicit description of the AS of the path P_n with n vertices. Firstly, we will prove that all the AS on P_n can be obtained from the Laplacian of P_m (for $m \leq n$) using an edge subdivision process. Using this fact, ballot sequences and lattice paths we will get that the number of AS on P_n obtained from the Laplacian of P_m is the ballot number

$$b(n-2, n-m) = \frac{m-1}{n-1}\binom{2n-2-m}{n-2}.$$

Therefore the number of the AS on P_n is the Catalan number C_{n-1}.

On the other hand, using a concept of extended AS we will present a way to generate the AS of P_n from a single extended AS, which leads to establish a bijection between the AS on P_n and the triangulations of a polygon. Therefore the extended AS of P_n exhibit an invariance under rotations. Using this, we will get that the number of AS of P_n with its i entry equal to $n-k-1$ is equal to the ballot number $b(n-2, k)$.