Given an action $\alpha$ of an inverse semigroup $S$ on a associative ring $A$ one may construct its associated skew inverse semigroup ring $A \rtimes_\alpha S$. We assume that $A$ is commutative and we define a certain commutative subring $T$ of $A \rtimes_\alpha S$ which coincides with the embedding of $A$ in $A \rtimes_\alpha S$ whenever $S$ is unital. Our main result asserts that $A \rtimes_\alpha S$ is a simple ring if, and only if, $T$ is a maximal commutative subring of $A \rtimes_\alpha S$ and $A$ is $S$-simple. As an application of our result we present a new proof of the simplicity criterion for a Steinberg algebra $A_R(G)$ associated with a Hausdorff and ample groupoid $G$. 