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Multiscale Hybrid-Mixed Method for Fluids

This work presents a family of finite element methods for multiscale fluid problems, named Multiscale Hybrid-Mixed (MHM) methods. The MHM method is a consequence of a hybridization procedure which characterizes the unknowns as a direct sum of a "coarse" solution and the solutions to problems with Neumann boundary conditions driven by the multipliers. As a result, the MHM method becomes a strategy that naturally incorporates multiple scales while providing solutions with high-order precision for the primal and dual variables. The completely independent local problems are embedded in the upscaling procedure, and computational approximations may be naturally obtained in a parallel computing environment. Well-posedness and best approximation results for the one- and two-level versions of the MHM method show that the method is optimal convergent and achieves super-convergence for the velocity field with respect to the mesh parameter. Interesting, the numerical velocity field turns out to be locally conservative. Also, a face-based a posteriori estimator is shown to be locally efficient and reliable with respect to the natural norms. The general framework and some recent results are illustrated for the Stokes and Brinkman equations, and validated through a large varieties of numerical results for highly heterogeneous coefficient problems.

References

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