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Gröbner bases of neural ideals

The brain processes information about the environment via neural codes. The neural ideal was introduced recently as an algebraic object that can be used to better understand the combinatorial structure of neural codes. Every neural ideal has a particular generating set, called the canonical form, that directly encodes a minimal description of the receptive field structure intrinsic to the neural code. On the other hand, for a given monomial order, any polynomial ideal is also generated by its unique (reduced) Gröbner basis with respect to that monomial order. How are these two types of generating sets – canonical forms and Gröbner bases – related? Our main result states that if the canonical form of a neural ideal is a Gröbner basis, then it is the universal Gröbner basis (that is, the union of all reduced Gröbner bases). Furthermore, we prove that this situation – when the canonical form is a Gröbner basis – occurs precisely when the universal Gröbner basis contains only pseudo-monomials (certain generalizations of monomials). Our results motivate two questions: (1) When is the canonical form a Gröbner basis? (2) When the universal Gröbner basis of a neural ideal is not a canonical form, what can the non-pseudo-monomial elements in the basis tell us about the receptive fields of the code? We give partial answers to both questions. Along the way, we develop a representation of pseudo-monomials as hypercubes in a Boolean lattice.