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Some algorithmic aspect of arithmetical structures

Let M be a non-negative $n \times n$ integer matrix with all the diagonal entries equal to zero. An arithmetical structure (AS) on M is a pair $(\mathbf{d}, \mathbf{r}) \in \mathbb{N}_+^n \times \mathbb{N}_+^n$ such that $\gcd(\mathbf{r}_v \,|\, v \in V(G)) = 1$ and

$$(\mathrm{diag}(\mathbf{d}) - M)\mathbf{r}^t = \mathbf{0}^t.$$

The concept of AS was introduced by Lorenzini as some intersection matrices that arise in the study of degenerating curves in algebraic geometry. If M is the adjacency matrix of a graph G and $\mathbf{d}$ is its degree vector, then $\mathrm{diag}(\mathbf{d}) - M$ is the Laplacian matrix of G. It can be proved that M is irreducible if and only if

$$\mathcal{A}(M) = \{(\mathbf{d}, \mathbf{r}) \in \mathbb{N}_+^n \times \mathbb{N}_+^n \,|\, (\mathrm{diag}(\mathbf{d}) - M)\mathbf{r}^t = \mathbf{0}^t\}$$

is finite. Recently an explicit description of the arithmetical structures of the path and cycle have been given. Let $X = (x_1, \ldots, x_n)$ be a vector with variables on each one of its entries and

$$f_M(X) = \det(\mathrm{diag}(X) - M).$$

It is not difficult to check that if $(\mathbf{d}, \mathbf{r}) \in \mathcal{A}(M)$, then $\mathbf{d}$ is a solution of the Diophantine equation $f_M(X) = 0$. Note that the converse is not true.

By Hilbert's tenth problem is not clear that an algorithm to compute the solutions of this class of Diophantine equations or even the arithmetical structures of M exist. In this talk we will present an algorithm to compute the AS on M.

Finally, if the time permits we will present some details of the implementation of this algorithm.

Joint work with Ralihe R. Villagran.