The construction of a functional integral representation for the solution of the Schrödinger equation, in other words the mathematical definition of "Feynman path integrals", requires an integration theory on infinite dimensional spaces which extends the Lebesgue one. In this talk I shall introduce a generalized approach to infinite dimensional integration which includes, on one hand, both probabilistic and oscillatory integrals and provides, on the other hand, the mathematical basis for the construction of generalized Feynman-Kac formulae. In particular it can be applied to the representation of the solution to partial differential equations which do not satisfy a maximum principle, such as, for instance, the Schrödinger equation or \( N \)-order heat-type equations of the form

\[
\begin{align*}
\frac{\partial}{\partial t} u(t, x) &= a \frac{\partial^N}{\partial x^N} u(t, x), \\
u(0, x) &= f(x)
\end{align*}
\]

where \( t \in \mathbb{R}^+, x \in \mathbb{R}, a \in \mathbb{C} \) and \( N \in \mathbb{N} \).