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The heat equation with the continuous primitive integral

A Schwartz distribution has a continuous primitive integral if it is the distributional derivative of a function that is continuous on the extended real line. This generalises the Lebesgue and Henstock–Kurzweil integrals. The Alexiewicz norm of f is $\|f\| = \sup |\int_I f|$ where the supremum is over all intervals $I \subset \mathbb{R}$. The space of distributions integrable in this sense is then a Banach space isometrically isomorphic to the continuous functions on the extended real line with the uniform norm. Many properties familiar from Lebesgue integration continue to hold for these distributions.

The one-dimensional heat equation is considered with initial data that is integrable in the sense of the continuous primitive integral. Let $\Theta_t(x) = \exp(-x^2/(4t))/\sqrt{4\pi t}$ be the heat kernel. With initial data f that is the distributional derivative of a continuous function, it is shown that $u_t(x) := u(x,t) := f * \Theta_t(x)$ is a classical solution of the heat equation $u_{11} = u_2$. The estimate $\|f * \Theta_t\|_\infty \leq \|f\|/\sqrt{\pi t}$ holds. The initial data is taken on in the Alexiewicz norm, $\|u_t - f\| \to 0$ as $t \to 0^+$. The solution of the heat equation is unique under the assumptions that $\|u_t\|$ is bounded and $u_t \to f$ in the Alexiewicz norm for some integrable f.