We investigate the solvability of the Ambrosetti-Prodi problem for the p-Laplace operator with Neumann boundary conditions. Using a priori estimates, regularity theory, a sub-supersolution method, and the Leray-Shauder degree theory, we obtain a necessary condition for the non-existence of solutions (in the weak sense), the existence of at least one solution, and the existence of at least two distinct solutions. Moreover, we establish global H"older continuity for weak solutions of the Neumann problem of Ambrosetti-Prodi type on a large class of non-smooth domains.