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Pair correlation statistics in subsets of the integers

Given $\mathcal{A} \subset \mathbb{N}$ and $\alpha \in \mathbb{R}$, it is often of interest to consider pair correlations of the set $\alpha\mathcal{A}$ and their distribution modulo 1. Denote by A_N the first N elements of \mathcal{A} . We say that \mathcal{A} is "metric Poissonian" if

$$\frac{1}{N} \sum_{\substack{a, b \in A_N \\ a \neq b}} 1_{[-s/N, s/N]}(\{\alpha(a-b)\}) \rightarrow 2s \quad \text{as } N \rightarrow \infty,$$

for almost all α and for all fixed s , where $\{x\} = x - \lfloor x \rfloor$ denotes the fractional part of x . The metric Poissonian property is a stronger notion of equidistribution modulo 1, and is closely related to the additive energy of the set. Indeed, Aistleitner, Larcher, and Lewko have shown that if the additive energy satisfies $E(A_N) = O(N^{3-\varepsilon})$ then \mathcal{A} is metric Poissonian. In an appendix to the same paper, Bourgain gives that \mathcal{A} cannot be metric Poissonian if $\limsup_{n \rightarrow \infty} E(A_n)N^{-3} > 0$. In this talk, we will discuss ways in which density and additive energy can be used to determine whether a set $\mathcal{A} \subset \mathbb{N}$ has the metric Poissonian property. This is joint work with Thomas Bloom, Sam Chow, and Aled Walker.